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ON STRESS REDISTRIBUTION IN STRUCTURES DURING CREEP

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Abstract—In the creep literature, the term "skeletal point" is often used to denote a point in a creeping body at which the stress is independent of time. It is shown for a class of stress redistribution problems involving one space dimension, both for secondary creep and strain-hardening primary creep, that such points do not exist. Also, for a one-dimensional continuum of bars whose configuration depends on a shape function $f(x)$, we show that, if f takes more than two distinct values, the relaxation times cannot be the same for all bars. However, if f takes at most two distinct values, the relaxation times will always be the same. This result assumes powerlaw secondary creep.

1. INTRODUCTION

A characteristic feature of many problems involving creeping metal structures subject to time-independent loads is the gradual redistribution of stresses from an initial elastic to an ultimate creep profile. This is generally modelled by representing the total strain as the sum of an elastic strain, which depends linearly on stresses, and an initially zero nonlinear creep strain. This paper presents results concerning two specific aspects of the redistribution phenomenon: skeletal points and the relaxation time.

Let $s(r, t)$ denote a one-dimensional stress profile (e.g. effective stress through the wall of a pressure vessel) for $a \leq r \leq b$ at time t. In general, if one plots the initial elastic stress $s(r, 0)$ (solid line) and the steady-state creep stress $s(r, \infty)$ (dotted line) on the same graph, patterns like those of Fig. 1 result. The point of cross-over, which occurs at $r = c$, was

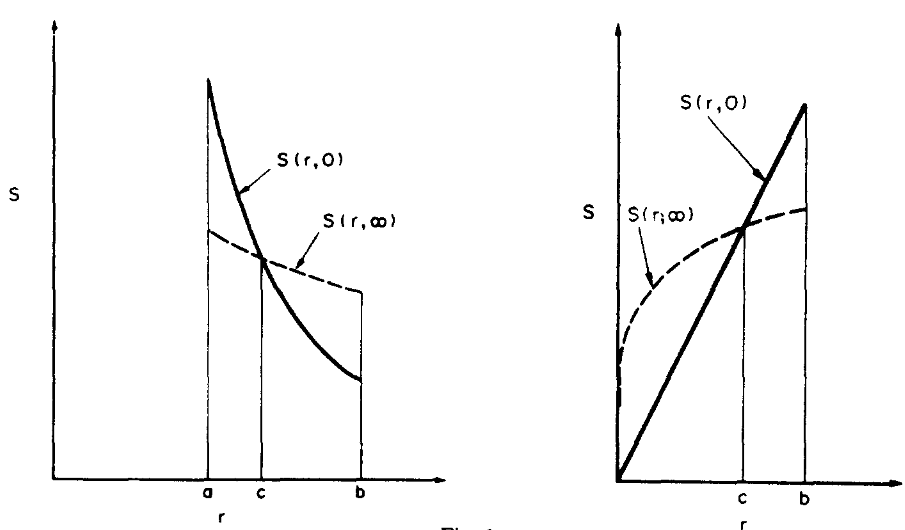

Fig. 1.

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named the "skeletal point" by Marriott and Leckie in a 1964 paper ([1], p. 119). Profiles $s(r, t)$ for $0 < t < \infty$ lie roughly between $s(r, 0)$ and $s(r, \infty)$ and appear to intersect one another at points very near to the skeletal point (see, e.g. Fig. 17.6d of [1]). In a 1970 paper ([2], pp. 146–148), Marriott described as an "engineering" assumption the hypothesis that the stress remains *constant* at the skeletal point for all time.

It appears that in later discussions of points of constant stress, the approximate nature of this phenomenon was overlooked. For example, in 1978, Goodman and Goodall[3] described the skeletal point as a position "where the stress level is insensitive to variation in the creep index n". Also, a 1980 textbook claimed that in [1], Marriott and Leckie had observed points in components undergoing transient creep "at which the stress does not change with time" ([4], p. 82).

In Section 2, we set up the equations on which our secondary creep work is based. Our first main result, established in Section 3, asserts that, for a large class of one dimensional stress distributions, skeletal points in the sense of the later authors, i.e. points at which the stress remains constant for all time, *do not exist*. We prove this both for power law secondary creep and strain-hardening primary creep. For the latter case, we develop a generalized primary creep equation analogous to (2.4) below.

Despite its nonexistence, the skeletal point remains a useful concept in making simple estimates of the creep behavior of various structural elements during the transition from initial to final steady state. It is beyond the scope of this paper to consider the magnitude of the error resulting from using the skeletal point concept in design calculations.

The relaxation time $\tau(r)$ estimates the time required for the stress at point r to realize its limiting value $s(r, \infty)$. Figure 2 motivates the definition† (see [5, 6])

$$\tau(r) \equiv \frac{s(r, \infty) - s(r, 0)}{\dot{s}(r, 0)}, \quad (a \leq r \leq b). \tag{1.1}$$

We are concerned here with relaxation times in a one dimensional continuum consisting of an infinite number of infinitely thin vertical bars which occupy the unshaded portion of Fig. 3. $L(x)$ is the initial length of the bar at position x. Its reciprocal $f(x) \equiv L^{-1}(x)$ is called the *shape function*. A question which arises naturally in the design of such a structure under creep conditions is whether a nonconstant shape function $f(x)$ exists such that all of the bars relax with the same relaxation time, i.e. such that $\tau(x) \equiv$ constant.

In Section 4, it is shown that if $\tau(x)$ is identically constant, then $f(x)$ *cannot take three distinct values*. Our proof, which holds for power law secondary creep, does not require any assumptions as to the smoothness of f. It is also shown that if f is any integrable step function which takes two distinct values, then $\tau(x)$ is identically constant.

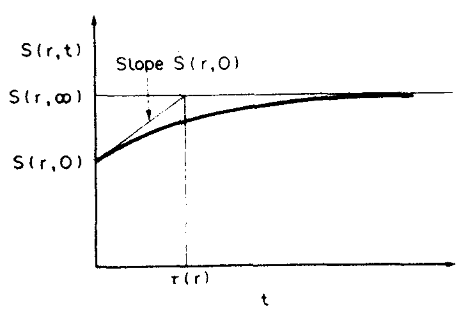

Fig. 2.

†The superposed dot stands for a time derivative.

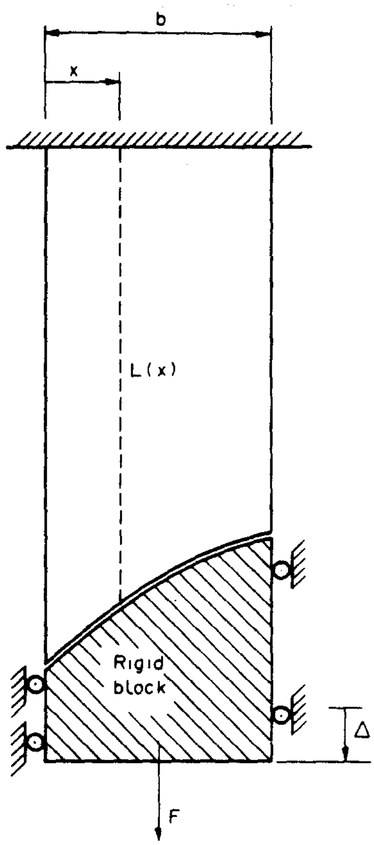

Fig. 3.

2. SECONDARY CREEP EQUATIONS

As shown in [7] transient creep states in a number of different components can be derived from the following non-linear integral equation

$$s(r,t)=\frac{r^l}{I}\left(N+\int_0^t\int_a^b H(s)q(\xi)\,d\xi\,d\tau\right)-\int_0^t H(s)\,d\tau \tag{2.1}$$

where $a \le r \le b$, $t \ge 0$ and

$$I=\int_a^b \xi^l q(\xi)\,d\xi \quad (q>0 \text{ on } (a,b)). \tag{2.2}$$

Here $s(r, t)$ represents a stress distribution, $N(t)$ an applied load and H a creep function, whereas q and l depend on the geometry of the component.

Equation (2.1) governs, as special cases, such problems as the torsion of a circular cylinder under moment M, taking

$$s=\frac{(1+\nu)}{E}\sigma_{\theta z},\quad l=1,\quad q(\xi)=\xi^2,\quad N=\frac{(1+\nu)M}{2\pi E},$$

and the pure bending of a symmetric beam[7], where

$$s=\frac{\sigma_{zz}}{E},\quad l=1,\quad N=-\frac{M}{E},\quad a=0,\quad b=c,\quad M<0.$$

Here, $q(\xi)$ is a positive integrable function depending on the shape of the beam's cross-section. Equation (2.1) also covers cylindrical ($l = -2$) and spherical ($l = -3$) pressure vessels, in which case it becomes (2.36) of [8], and N is proportional to the internal pressure.

It is clear in all cases that s is proportional to stress and that N is a known quantity related to the applied load. In the problems discussed below, N is a positive constant. With suitable assumptions on the creep function H, one can then prove that s must be positive. In the present work, we shall generalize by replacing r^l by the positive, piecewise continuous *nonconstant* function $f(r)$ and specialize by taking†

$$H(s) = Ks^n, \quad (K > 0,\ n > 1). \tag{2.3}$$

Equations (2.1) and (2.2) now become

$$s(r, t) = \frac{f(r)N}{I} + K\int_0^t \left(\frac{f(r)}{I}\int_a^b s^n(\xi, \tau)q(\xi)\,\mathrm{d}\xi - s^n(r, \tau)\right)\mathrm{d}\tau, \tag{2.4}$$

$$I = \int_a^b f(\xi)q(\xi)\,\mathrm{d}\xi. \tag{2.5}$$

Setting $t = 0$, we get the initial elastic stress

$$s(r, 0) = \frac{f(r)N}{I}. \tag{2.6}$$

It is easy to see that for $n = 1$, $s(r, t)$ takes the value (2.6) for all $t > 0$. The steady-state creep profile $s(r, \infty)$ can be obtained as follows, see [8]. Differentiation of equation (2.4) with respect to time yields

$$\dot{s}(r, t) = \frac{f(r)\dot{N}}{I} + K\left(\frac{f(r)}{I}\int_a^b s^n(\xi, t)q(\xi)\,\mathrm{d}\xi - s^n(r, t)\right). \tag{2.7}$$

With $\dot{s}(r, \infty) = 0$ and $\dot{N} = 0$ follows

$$s^n(r, \infty) = \frac{f(r)}{I}\int_a^b s^n(\xi, \infty)q(\xi)\,\mathrm{d}\xi \tag{2.8}$$

and hence

$$s(r, \infty) = Cf^{1/n}(r) \tag{2.9}$$

with C a constant, which may be determined as shown in [8], p. 273. The resultant steady-state profile is

$$s(r, \infty) = \frac{f^{1/n}(r)N}{\displaystyle\int_a^b f^{1/n}(\xi)q(\xi)\,\mathrm{d}\xi}. \tag{2.10}$$

The proof of nonexistence of skeletal points for secondary creep will be based on (2.4).

For the continuous distribution of bars shown in Fig. 3, the tensile stress $\sigma(x, t)$ and strain $\epsilon(x, t)$ satisfy

$$\int_0^b \sigma(x, t)h\,\mathrm{d}x = F \text{ (equilibrium)}, \tag{2.11}$$

†For more general loading histories $N(t)$, under which s could take positive or negative values, (2.3) would be replaced by $H(s) = K|s|^{n-1}s$.

$$\epsilon(x,t) = \frac{\Delta(t)}{L(x)} = \Delta(t)f(x) \text{ (compatibility)} \tag{2.12}$$

and the creep law

$$\epsilon(x,t) = E^{-1}\sigma(x,t) + B\int_0^t \sigma^n(x,\tau)\,d\tau, \tag{2.13}$$

with

$$E > 0, \quad B > 0, \quad n > 1.$$

In this notation, the relaxation time becomes

$$\tau(x) = \frac{\sigma(x,\infty) - \sigma(x,0)}{\dot{\sigma}(x,0)} \quad (0 \le x \le b). \tag{2.14}$$

Equations (2.11)–(2.13) immediately imply an equation in σ of the form (2.4). In fact, substitution of (2.13) into (2.12) yields

$$\Delta(t)f(x) = E^{-1}\sigma(x,t) + B\int_0^t \sigma^n(x,\tau)\,d\tau. \tag{2.15}$$

If we then integrate (2.15) with respect to x from 0 to b and apply (2.11), we get

$$\Delta(t) = \frac{1}{I}\left(\frac{F}{Eh} + B\int_0^t\int_0^b \sigma^n\,d\xi\,d\tau\right), \tag{2.16}$$

$$I = \int_0^b f(\xi)\,d\xi. \tag{2.17}$$

Substitution of (2.16) into (2.15) furnishes the desired equation

$$\sigma(x,t) = \frac{f(x)F}{Ih} + EB\int_0^t\left(\frac{f(x)}{I}\int_0^b \sigma^n(\xi,\tau)\,d\xi - \sigma^n(x,\tau)\right)d\tau, \tag{2.18}$$

which clearly is included in (2.4). Thus, our result on nonexistence of skeletal points also holds for the continuum of bars. Furthermore, it is easily seen that for suitably chosen $f(x)$, the results of [7] on upper and lower sequences of approximations can be extended to this problem.

3. NONEXISTENCE OF SKELETAL POINTS

Suppose at least one skeletal point exists for some c in the interval $[a,b]$, i.e.

$$s(c,t) \equiv \text{constant for all } t \ge 0. \tag{3.1}$$

Then, equating (2.6) and (2.7) for $r = c$, we get

$$f^{n-1}(c) = \frac{\left[\int_a^b fq\,d\xi\right]^n}{\left[\int_a^b f^{1/n}q\,d\xi\right]^n}. \tag{3.2}$$

On the other hand, one time differentiation of (2.4) yields

$$\frac{\dot{s}(r,t)}{K} = \frac{f(r)}{I}\int_a^b s^n(\xi,t)q(\xi)\,d\xi - s^n(r,t).$$

Now let $r = c$. Then, since $\dot{s}(c, t) = 0$,

$$f(c) = \frac{Is^n(c, t)}{\int_a^b s^n(\xi, t)q(\xi)\,\mathrm{d}\xi}. \tag{3.3}$$

Since this equation holds for all t, we can, in particular, set $t = 0$ and then substitute (2.6) into it. The result is

$$f^{n-1}(c) = \frac{\int_a^b f^n q\,\mathrm{d}\xi}{\int_a^b fq\,\mathrm{d}\xi}. \tag{3.4}$$

Eliminating $f^{n-1}(c)$ between (3.2) and (3.4), we get

$$\int_a^b fq\,\mathrm{d}\xi = \left[\int_a^b f^n q\,\mathrm{d}\xi\right]^{1/(n+1)} \cdot \left[\int_a^b f^{1/n} q\,\mathrm{d}\xi\right]^{n/(n+1)}. \tag{3.5}$$

With

$$f^n \equiv f_1^{n+1}, \quad f^{1/n} \equiv f_2^{(n+1)/n}, \tag{3.6}$$

so that

$$f = f^{n/(n+1)} \cdot f^{1/(n+1)} = f_1 \cdot f_2,$$

(3.5) becomes

$$\int_a^b f_1 f_2 q\,\mathrm{d}\xi = \left[\int_a^b f_1^{n+1} q\,\mathrm{d}\xi\right]^{1/(n+1)} \cdot \left[\int_a^b f_2^{(n-1)/n} q\,\mathrm{d}\xi\right]^{n/(n+1)} \tag{3.7}$$

Holder's inequality with weight function q states[9] that, for sufficiently smooth functions f_1, f_2,

$$\left|\int_a^b f_1 f_2 q\,\mathrm{d}\xi\right| \leq \left[\int_a^b |f_1|^{p_1} q\,\mathrm{d}\xi\right]^{1/(p_1)} \cdot \left[\int_a^b |f_2|^{p_2} q\,\mathrm{d}\xi\right]^{1/(p_2)} \tag{3.8}$$

for any $p_1 > 1$, $p_2 > 1$ such that $1/p_1 + 1/p_2 = 1$. Moreover, equality holds only if there exists a nonzero constant λ such that

$$|f_1(r)|^{p_1} \equiv \lambda |f_2(r)|^{p_2} \text{ in } [a, b].$$

Applying this to the positive functions f_1, f_2 defined by (3.6) with $p_1 = n + 1$ and $p_2 = (n + 1)/n$, we see that (3.7) implies

$$f^n(r) \equiv \lambda f^{1/n}(r) \text{ in } [a, b].$$

Since for $n > 1$, this contradicts the condition that f be nonconstant, the nonexistence of skeletal points is thus established. Recall that for such problems as the pure bending of beams, torsion of circular cylinders, etc. $f(r)$ had the form r^l where l is a non-zero constant. For the continuum of bars, a nonconstant shape function $f(x)$ was assumed, since for f constant, a degenerate case results.

We now consider the extension of this result to bodies undergoing strain-hardening creep according to the creep law

$$\dot{\epsilon}_{ij}^{(c)} = \frac{3K\sigma_e^{n-1}}{2[\epsilon_e^{(c)}]^m} \cdot s_{ij}(t > 0), \quad \epsilon_{ij}^{(c)}\big|_{t=0} = 0 \tag{3.9}$$

of Odqvist and Hult[10]. Here $\sigma_e, \epsilon_e^{(c)}$ stand for the effective stress and the effective creep strain respectively and s_{ij} for the deviatoric components of the stress. It has been shown[7] that primary creep both in circular bars undergoing torsion and symmetric beams subject to pure bending is governed by the equation (compare (2.8) of [7] and subsequent remarks)

$$\sigma = \frac{rN}{I} + \frac{rB}{I}\int_a^b \left[\int_0^t \sigma^n \,\mathrm{d}\tau\right]^{1/(m+1)} q(\xi)\,\mathrm{d}\xi - B\left[\int_0^t \sigma^n \,\mathrm{d}\tau\right]^{1/(m+1)}. \tag{3.10}$$

The constant B depends on which case is being considered, as does the interpretation of σ.

Also, in [11] it was shown that, with the same strain-hardening law, primary creep in internally loaded symmetric pressure vessels reduces to the equation

$$\sigma = \frac{r^{-j}N}{I} + \frac{r^{-j}B}{I}\int_a^b \left[\int_0^t \sigma^n \,\mathrm{d}\tau\right]^{1/(m+1)} \frac{\mathrm{d}\xi}{\xi} - B\left[\int_0^t \sigma^n \,\mathrm{d}\tau\right]^{1/(m+1)} (0 < a < b). \tag{3.11}$$

In fact, this is just (2.34) of [11] with the notation modified in an obvious way. Both (3.10) and (3.11) are contained in

$$s(r,t) = \frac{f(r)N}{I} + \frac{f(r)B}{I}\int_a^b \left[\int_0^t s^n \,\mathrm{d}\tau\right]^{1/(m+1)} q(\xi)\,\mathrm{d}\xi - B\left[\int_0^t s^n \,\mathrm{d}\tau\right]^{1/(m+1)}, \tag{3.12}$$

$$I = \int_a^b f(\xi)q(\xi)\,\mathrm{d}\xi. \tag{3.13}$$

We assume that N is a positive constant, that

$$q(\xi) > 0 (a < \xi < b), \quad n > 1, \quad m + 1 < n, \quad B > 0$$

and f is positive and nonconstant. Also it must be assumed that s is positive and continuous in $[a, b] \times [0, \infty)$. It follows from (3.12) that

$$s(r, 0) = \frac{f(r)N}{I} \tag{3.14}$$

as in the case of secondary creep. As $t \to \infty$, one can formally deduce from (3.12), using the technique of Section 4 of [11] that

$$s(r, \infty) = \frac{Nf^{1/\alpha}(r)}{\int_a^b f^{1/\alpha}(\xi)q(\xi)\,\mathrm{d}\xi}, \quad \alpha = \frac{n}{m+1} > 1. \tag{3.15}$$

Again, suppose that a skeletal point exists at $r = c$. Then, since $s(c, 0) = s(c, \infty)$, (3.14) and (3.15) imply

$$f^{\alpha-1}(c) = \frac{\left[\int_a^b fq \,\mathrm{d}\xi\right]^\alpha}{\left[\int_a^b f^{1/\alpha}q \,\mathrm{d}\xi\right]^\alpha} \tag{3.16}$$

in analogy to (3.2). For the derivation of the primary creep analogue of (3.4) we again differentiate the basic integral equation, in this case (3.12), with respect to time at $r = c$ to get

$$\frac{f(c)}{I}\int_a^b \left[\int_0^t s^n \,\mathrm{d}\tau\right]^{[-m/(m+1)]} s^n(\xi, t)q(\xi)\,\mathrm{d}\xi = \left[\int_0^t s^n(c, \tau)\,\mathrm{d}\tau\right]^{[-m/(m+1)]} s^n(c, t). \tag{3.17}$$

Since

$$\lim_{t\to 0}\frac{t}{\int_0^t s^n(r,\tau)\,d\tau}=\frac{1}{s^n(r,0)},$$

we find, upon multiplying both sides of (3.17) by $t^{m/(m+1)}$ and taking the limit as $t\to 0$ that

$$\frac{f(c)}{I}\int_a^b s^\alpha(\xi,0)q(\xi)\,d\xi = s^\alpha(c,0).$$

This has the same form as (3.3) with $t=0$ and n replaced by α. The rest of the argument is the same as for the secondary creep case.

4. SHAPE FUNCTIONS AND RELAXATION TIMES

In order to compute the relaxation time $\tau(x)$ at fiber x from (2.12), we use (2.15), (2.16) to obtain

$$\sigma(x,0)=\frac{f(x)F}{Ih},\quad I=\int_0^b f(\xi)\,d\xi, \tag{4.1}$$

$$\dot\sigma(x,0)=EB\left(\frac{F}{Ih}\right)^n\left(\frac{f(x)}{I}\int_0^b f^n(\xi)\,d\xi - f^n(x)\right). \tag{4.2}$$

Comparing (2.16) with (2.4), we see that, for the continuum of bars, (2.7) becomes

$$\sigma(x,\infty)=\frac{f^{1/n}(x)F}{h\int_0^b f^{1/n}(\xi)\,d\xi}. \tag{4.3}$$

Substitution of (4.1)–(4.3) into (2.12) now yields

$$\tau(x)=\frac{(F/h)^{1-n}I^n}{EB\left(\int f^n\,d\xi\right)}\cdot\frac{\dfrac{f^{1/n}(x)}{\int f^{1/n}\,d\xi}-\dfrac{f(x)}{I}}{\dfrac{f(x)}{I}-\dfrac{f^n(x)}{\int f^n\,d\xi}}. \tag{4.4}$$

Suppose $\tau(x)$ is identically constant on $[0,b]$, and let there exist points x_1, x_2, x_3 in $[0,b]$ (in any order) with $f(x_i)\equiv c_i$ $(i=1,2,3)$ such that $c_i\neq c_j$ for $i\neq j$, $\min c_i>0$. The latter follows from the assumption that $f>0$ on $[a,b]$. Then, if (4.4) is evaluated at the three points x_1, x_2, x_3, we get

$$\lambda=\frac{\dfrac{c_i^{1/n}}{\int f^{1/n}\,d\xi}-\dfrac{c_i}{\int f\,d\xi}}{\dfrac{c_i}{\int f\,d\xi}-\dfrac{c_i^n}{\int f^n\,d\xi}},\quad (i=1,2,3)$$

where λ is independent of i. That is,

$$(1+\lambda)\frac{c_i}{\int f\,d\xi}=\frac{c_i^{1/n}}{\int f^{1/n}\,d\xi}+\frac{\lambda}{\int f^n\,d\xi}c_i^n. \tag{4.5}$$

Obviously, (4.5) has the form

$$\alpha c_i^{1/n} + \beta c_i + \gamma c_i^n = 0 \quad (i = 1, 2, 3). \tag{4.6}$$

Since $|\alpha| + |\beta| + |\gamma| > 0$, the determinant Δ of the 3 by 3 linear system (4.6) must equal zero. But

$$\Delta = \begin{vmatrix} c_1^{1/n} & c_1 & c_1^n \\ c_2^{1/n} & c_2 & c_2^n \\ c_3^{1/n} & c_3 & c_3^n \end{vmatrix} = [c_1c_2c_3]^{1/n} \begin{vmatrix} 1 & c_1^{(n-1)/n} & [c_1^{(n-1)/n}]^{n+1} \\ 1 & c_2^{(n-1)/n} & [c_2^{(n-1)/n}]^{n+1} \\ 1 & c_3^{(n-1)/n} & [c_3^{(n-1)/n}]^{n+1} \end{vmatrix}.$$

Let $a_i = c_i^{(n-1)/n}$. Then for $n > 1$, $a_i \neq a_j$ for $i \neq j$, and

$$\Delta = [c_1c_2c_3]^{1/n} \begin{vmatrix} 1 & a_1 & a_1^{n+1} \\ 1 & a_2 & a_2^{n+1} \\ 1 & a_3 & a_3^{n+1} \end{vmatrix}.$$

Recall that (see, e.g. [12], p. 338) three points (x_1, y_1), (x_2, y_2) and (x_3, y_3) can lie on a straight line if and only if

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0.$$

But, for $n > 1$, the three points (a_i, a_i^{n+1}) cannot lie on a straight line due to the strict convexity of the function $y = x^{n+1}$. We have thus proved that if $\tau(x)$ *is identically constant on* $[a, b]$, *then* $f(x)$ *cannot take three distinct values.*

Now suppose that f is an integrable function which takes two distinct values, say c_1 and c_2, on $[a, b]$. Let b_i be the measure of the subset of $[0, b]$ on which $f = c_i$. If f is a classical step function, then b_i is simply the sum of the lengths of the intervals on which $f = c_i$. Let

$$Q_i = \frac{\dfrac{c_i^{1/n}}{\int f^{1/n}\,d\xi} - \dfrac{c_i}{\int f\,d\xi}}{\dfrac{c_i}{\int f\,d\xi} - \dfrac{c_i^n}{\int f^n\,d\xi}} \quad (i = 1, 2) = \frac{\dfrac{c_i^{1/n}}{b_1c_1^{1/n} + b_2c_2^{1/n}} - \dfrac{c_i}{b_1c_1 + b_2c_2}}{\dfrac{c_i}{b_1c_1 + b_2c_2} - \dfrac{c_i^n}{b_1c_1^n + b_2c_2^n}}. \tag{4.7}$$

Let τ_i be the value taken by τ at those points x at which $f(x) = c_i$. In order to prove τ identically constant as asserted in Section 1, suffices to show that $\tau_1 = \tau_2$. A glance at (4.4) tells us that this will follow once it is seen that $Q_1 = Q_2$. But, by (4.7),

$$Q_1 = Q_2 = \frac{(b_1c_1^n + b_2c_2^n)(c_2^{1/n}c_1 - c_2c_1^{1/n})}{(b_1c_1^{1/n} + b_2c_2^{1/n})(c_2c_1^n - c_2^nc_1)}.$$

CONCLUSION

Detailed studies of the stress redistribution, which occurs due to nonlinear creep in structure elements subject to time independent loads, have shown that certain common features exist. One of these is the existence of a small region within the structure, where the stress level remains nearly constant throughout the redistribution process. Various authors have postulated the existence of a point, where the stress is constant, and termed this the "skeletal point". It has been shown in the present paper that, for a wide class of structural elements of engineering interest, such points do not exist. Hence design methods based on the concept of a skeletal point are only approximate methods. No attempt has been made here to estimate the accuracy obtainable in using skeletal point methods.

For the related problem of determining the relaxation times in an array of parallel bars of varying length it has been shown that the relaxation time cannot be equal for all bars except for the trivial case of two sets of bars, all the bars in each set having the same length, when equality always holds.

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